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SUSY breaking and moduli stabilization from fluxes in gauged $6D$ supergravity

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Abstract: We construct the 4D $N = 1$ supergravity which describes the low-energy limit of $6D$ supergravity compactified on a sphere with a monopole background *à la* Salam and Sezgin. This provides a simple setting sharing the main properties of realistic string compactifications such as flat 4D spacetime, chiral fermions and $N = 1$ supersymmetry as well as Fayet-Iliopoulos terms induced by the Green-Schwarz mechanism. The matter content of the resulting theory is a supersymmetric SO(3) $\times$ U(1) gauge model with two chiral multiplets, S and T. The expectation value of T is fixed by the classical potential, and S describes a flat direction to all orders in perturbation theory. We consider possible perturbative corrections to the Kähler potential in inverse powers of ReS and ReT, and find that under certain circumstances, and when taken together with low-energy gaugino condensation, these can lift the degeneracy of the flat direction for ReS. The resulting vacuum breaks supersymmetry at moderately low energies in comparison with the compactification scale, with positive cosmological constant. It is argued that the $6D$ model might itself be obtained from string compactifications, giving rise to realistic string compactifications on non Ricci flat manifolds. Possible phenomenological and cosmological applications are briefly discussed.

Keywords: Supersymmetry Breaking, Supersymmetry Phenomenology, Supergravity Models, Compactification and String Models.

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1. Introduction

Realistic string models in $4D$ share many interesting properties. They correspond to backgrounds for which the spacetime is $4D$ Minkowski times an internal $6D$ manifold, and they have chiral fermions in $4D$ with spectrum close to the Standard Model. They also have important unsolved issues such as supersymmetry breaking and the presence of moduli fields such as the dilaton [1]. Several ideas have been put forward to deal with these issues, including nonperturbative effects, such as gaugino condensation [2], and the introduction of fluxes of antisymmetric tensor fields [3]. Furthermore brane/antibrane systems and intersecting branes have been considered both to obtain realistic models with broken supersymmetry [4] and for the possibility of generating cosmological inflation [5, 6].

However, there is at the moment not a single model that can achieve all the successes simultaneously. For instance, to get inflation it is needed to assume that some of the moduli have been fixed by an unknown mechanism. Fluxes of Ramond-Ramond fields have been used to fix some moduli but not all of them. Supersymmetric models have to face the breaking of supersymmetry, and gaugino condensation and other nonperturbatively generated superpotentials usually lead to runaway potentials [2, 7]. Non-supersymmetric models such as brane/antibrane systems at singularities or intersecting brane models tend to be unstable, with the corresponding scalar potential not under control.

In this article we start an investigation of most of these issues in a setting that is much simpler than the explicit string constructions and yet shares the relevant properties of those models. The starting point is minimal $6D$ gauged supergravity coupled to at least one U(1) vector multiplet. This model was studied in [10], who considered compactification on a two-sphere stabilized by a nonvanishing magnetic flux through the sphere for the U(1) gauge field. This compactification was found to lead to a chiral $N = 1$ supersymmetric model in flat $4D$ spacetime at scales lower than the compactification scale. We here reconsider this model and study its consequences in more detail. We may see this as a first attempt to extract phenomenological implications to gauged supergravity potentials that are being derived recently in string theory. In this article we concentrate only on general issues concerning the model's low-energy effective action, supersymmetry breaking and moduli stabilization. We investigate how the introduction of branes changes the implications of this 'bulk' physics in a subsequent publication.

The model has several stringy properties, such as the presence of the standard S and T moduli fields of string compactifications, and we use these to address the issue of lifting the vacuum degeneracy. Furthermore, the low-energy theory has a Fayet-Iliopoulos term which is generated by the Green-Schwarz anomaly-cancelling mechanism. We find this induces a D-term potential in the low-energy theory that can naturally fix the field T, leaving only the S direction flat to all orders in perturbation theory.

The nonabelian SO(3) symmetry associated to the isometries of the two-sphere is asymptotically free and so generates a nonperturbative potential for S through the gaugino condensation mechanism. Together with the tree-level Kähler potential, this leads to a runaway potential for the dilaton field S. This kind of scenario, with fixed T and with runaway S, could provide a natural way for the model to realize inflation, along the lines suggested in ref. [6]. We further argue that if certain perturbative corrections to the Kähler potential arise, then the gaugino-condensation potential need not generate a runaway, and could be used to stabilize S. As is expected on general grounds, this minimum arises at the margins of what can be computed using semiclassical methods.

We organize our presentation as follows. The next section gives a brief review of the relevant features of $6D$ gauged supergravity, and its supersymmetric compactification to $4D$ on a sphere. Section (3) then derives the low-energy $4D$ supergravity which describes the low-energy limit of this compactification, as well as discussing its likely vacuum. The flat directions of the low-energy theory are the topic of Section (4), where it is shown that all are lifted. This section shows that some moduli are stabilized at finite values, while others may be stabilized, or run to infinity, depending on the details of the corrections to the model's Kähler function. These results, and some of their potential applications, are discussed in section (5), where we also point out that the $6D$ model itself may be derived from compactifications of string theory, either on sphere or K_3 compactifications in the presence of fluxes of antisymmetric tensor fields.

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2. The $6D$ Salam-Sezgin model

We begin by recapping the Salam-Sezgin compactification of the six-dimensional supersymmetric Einstein-Maxwell system [8]–[10].

2.1 The model

The field content of the theory consists of a supergravity multiplet — which comprises a metric (g_{MN}), antisymmetric Kalb-Ramond field (B_{MN}), dilaton (ϕ), gravitino (ψ_M) and dilatino (χ) — coupled to a U(1) gauge multiplet — containing a gauge potential (A_M) and gaugino (λ).

The fermions are all complex Weyl spinors — satisfying $\Gamma_7\psi_M = \psi_M$, $\Gamma_7\lambda = \lambda$ and $\Gamma_7\chi = -\chi$ — and they all transform under the U(1) gauge symmetry. For instance, the gravitino covariant derivative is

$$D_M\psi_N = \left(\partial_M - \frac{1}{4}\omega_M{}^{AB}\Gamma_{AB} - igA_M\right)\psi_N\,, \tag{2.1}$$

where $\omega_M{}^{AB}$ denotes the spin connection. Here g denotes the $6D$ U(1) gauge coupling, which in fundamental units ($\hbar = c = 1$) has the dimension $(\text{mass})^{-1}$.

The field strength for B_{MN} contains the usual supergravity Chern-Simons contribution

$$G_{MNP} = \partial_M B_{NP} + F_{MN}A_P + (\text{cyclic permutations})\,, \tag{2.2}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the usual abelian gauge field strength. Notice that the appearance of A_M in this equation implies B_{MN} must also transform under the U(1) gauge transformations, since invariance of G_{MNP} requires

$$\delta A_M = \partial_M\omega\,, \qquad \delta B_{MN} = -\omega\, F_{MN}\,. \tag{2.3}$$

This transformation allows the gauge anomalies due to the chiral fermions to be cancelled by a Green-Schwarz mechanism, as must happen if this supergravity emerges as a low-energy compactification of string theory (more about this below).

The bosonic part of the classical $6D$ supergravity action is:[1]

$$e^{-1}\mathcal{L}_B = -\frac{1}{2}R - \frac{1}{2}\partial_M\phi\,\partial^M\phi - \frac{e^{-2\phi}}{12}\,G_{MNP}G^{MNP} - \frac{e^{-\phi}}{4}\,F_{MN}F^{MN} - 2g^2e^{\phi}\,, \tag{2.4}$$

where we choose units for which the $6D$ Planck mass is unity: $\kappa_6^2 = 8\pi G_6 = 1$. As usual $e = |\det e_M{}^A| = \sqrt{-\det g_{MN}}$.

The part of the action which is bilinear in the fermions is

$$\begin{aligned}
e^{-1}\mathcal{L}_F =\; & -\bar\psi_M\Gamma^{MNP}D_N\psi_P - \bar\chi\Gamma^M D_M\chi - \bar\lambda\Gamma^M D_M\lambda - \\
& -\frac{1}{2}\partial_M\phi\left(\bar\chi\Gamma^N\Gamma^M\psi_N + \bar\psi_N\Gamma^M\Gamma^N\chi\right) + \\
& +\frac{e^{-\phi}}{12\sqrt{2}}\,G_{MNP}\Big(-\bar\psi^R\Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\psi^S + \bar\psi_R\Gamma^{MNP}\Gamma^R\chi - \\
& \qquad\qquad - \bar\chi\Gamma^R\Gamma^{MNP}\psi_R + \bar\chi\Gamma^{MNP}\chi - \bar\lambda\Gamma^{MNP}\lambda\Big) - \\
& -\frac{e^{-\phi/2}}{4}\,F_{MN}\left(\bar\psi_Q\Gamma^{MN}\Gamma^Q\lambda + \bar\lambda\Gamma^Q\Gamma^{MN}\psi_Q - \bar\chi\Gamma^{MN}\lambda + \bar\lambda\Gamma^{MN}\chi\right) + \\
& +ige^{\phi/2}\left(\bar\psi_M\Gamma^M\lambda + \bar\lambda\Gamma^M\psi_M + \bar\chi\lambda - \bar\lambda\chi\right),
\end{aligned} \tag{2.5}$$

where the completely antisymmetric products of Dirac matrices are defined by $\Gamma_{MN} = \frac{1}{2}\left(\Gamma_M\Gamma_N - \Gamma_N\Gamma_M\right)$, $\Gamma_{MNP} = \frac{1}{6}\left(\Gamma_M\Gamma_N\Gamma_P \pm \text{permutations}\right)$ and so on.

The $6D$ supersymmetry transformations which preserve the form of this action are

$$\begin{aligned}
\delta e_M^A &= \frac{1}{\sqrt{2}}\left(\bar\epsilon\Gamma^A\psi_M - \bar\psi_M\Gamma^A\epsilon\right) \\
\delta\phi &= -\frac{1}{\sqrt{2}}\left(\bar\epsilon\chi + \bar\chi\epsilon\right) \\
\delta B_{MN} &= \sqrt{2}\,A_{[M}\delta A_{N]} + \frac{e^{\phi}}{2}\Big(\bar\epsilon\Gamma_M\psi_N - \bar\psi_N\Gamma_M\epsilon - \\
&\qquad - \bar\epsilon\Gamma_N\psi_M + \bar\psi_M\Gamma_N\epsilon - \bar\epsilon\Gamma_{MN}\chi + \bar\chi\Gamma_{MN}\epsilon\Big)
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
\delta\chi &= \frac{1}{\sqrt{2}}\,\partial_M\phi\,\Gamma^M\epsilon + \frac{e^{-\phi}}{12}G_{MNP}\,\Gamma^{MNP}\epsilon \\
\delta\psi_M &= \sqrt{2}\,D_M\epsilon + \frac{e^{-\phi}}{24}\,G_{PQR}\,\Gamma^{PQR}\Gamma_M\epsilon \\
\delta A_M &= \frac{1}{\sqrt{2}}\left(\bar\epsilon\Gamma_M\lambda - \bar\lambda\Gamma_M\epsilon\right)e^{\phi/2} \\
\delta\lambda &= \frac{e^{-\phi/2}}{4}\,F_{MN}\,\Gamma^{MN}\epsilon - \frac{i}{\sqrt{2}}\,g\,e^{\phi/2}\,\epsilon\,,
\end{aligned} \tag{2.7}$$

where the supersymmetry parameter is complex and Weyl: $\Gamma_7\epsilon = \epsilon$.

[1] Our metric is 'mostly plus' and like all right-thinking people we follow Weinberg's curvature conventions [11].

2.1.1 Global and approximate symmetries

Besides supersymmetry, the U(1) gauge symmetry and the Kalb-Ramond symmetry, $\delta B = d\Lambda$, the model also has a few other symmetries (and approximate symmetries) which are useful to enumerate here for later convenience.

First, if the background spacetime admits an harmonic 2-form, Ω, then because B only enters the action through dB the background has a global symmetry $\delta B = c\,\Omega$, where c is the constant symmetry parameter. Because $\Omega \neq d\Lambda$ for any globally-defined 1-form this symmetry can be regarded as independent of the Kalb-Ramond gauge symmetry.

Second, the field equations obtained from this action have a classical symmetry under the following constant rescaling of the fields:

$$g_{MN} \to \sigma\, g_{MN}\,, \qquad e^{\phi} \to \frac{e^{\phi}}{\sigma}\,, \qquad \psi_M \to \sigma^{1/4}\,\psi_M\,, \qquad \chi \to \frac{\chi}{\sigma^{1/4}}\,, \qquad \lambda \to \frac{\lambda}{\sigma^{1/4}}\,, \tag{2.8}$$

with no other fields transforming. This is just a symmetry of the equations of motion, rather than a *bona fide* symmetry because it does not leave the action invariant, but rather rescales it according to $\mathcal{L} \to \sigma^2 \mathcal{L}$. This symmetry reflects the possibility of performing redefinitions to write the lagrangian density as $\mathcal{L} = e^{-2\phi}\,\mathcal{L}_{\rm inv}$, with $\mathcal{L}_{\rm inv}$ a function only of the invariant 'string-frame' quantities $g^s_{MN} = e^{\phi}\,g_{MN}$, $\psi^s_M = e^{\phi/4}\,\psi_M$, $\chi^s = e^{-\phi/4}\,\chi$, $\lambda^s = e^{-\phi/4}\,\lambda$ and $\partial_M\phi$. In general, since only the dilaton transforms under the scaling symmetry in the string frame, the ℓ-string-loop contribution to the action scales as $\mathcal{L}_\ell \to \sigma^{2-2\ell}\,\mathcal{L}_\ell$.

2.2 Anomaly cancellation

As mentioned above, the fermion content of the $6D$ model as described so far has anomalies, which must be cancelled if the theory is to make physical sense. In particular, they must cancel if the model is to be considered as the low-energy limit of an underlying consistent theory, such as string theory, at still higher energies. The anomaly cancellation conditions for $6D$ supergravity coupled to a single tensor- plus n_V vector- and n_H hyper-multiplets are well understood, so we simply summarize here several features which are used below.

These anomaly-cancelling conditions are significant for two separate reasons. First, they provide new nontrivial constraints on the kinds of particles which must appear in the $6D$ theory. In particular they require the existence of many more $6D$ matter supermultiplets than are considered above. Second, they show the necessity for specific higher-derivative corrections to the above supergravity lagrangian without reference to any specific, more microscopic, theory like string theory. As we shall see, the neglect of these corrections when using the above $6D$ supergravity lagrangian ultimately requires for consistency the conditions

$$\frac{1}{r^2} \ll e^{\phi} \ll 1\,, \tag{2.9}$$

with r defined in terms of the volume of the extra dimensions using the Einstein-frame metric.

2.2.1 The $6D$ Green-Schwarz mechanism

Six-dimensional anomalies are described by an 8-form constructed from the gauge and gravitational field strengths [12]. In order for anomalies to be cancelled by a Green-Schwarz

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type mechanism [13] — involving the shifting of a bosonic field in the theory — this anomaly form must factorize into the wedge product of pairs of lower-dimension forms.[2] In general this imposes a strong set of conditions in six dimensions, some features of which we now summarize [14, 15].

To describe the anomaly cancellation conditions we must first generalize the above field content to potentially include n_V gauge multiplets, as well as n_H matter 'hyper-' multiplets which involve $6D$ scalars and fermions whose helicity satisfies $\Gamma_7 = -1$. $6D$ supersymmetry requires the scalars within these hypermultiplets to take values in a quaternionic manifold, and precludes them from appearing in the gauge kinetic terms or in the kinetic term for the dilaton field ϕ [17].

A necessary condition for the factorizing of the anomaly 8-form is the vanishing of the coefficient of the $\mathrm{tr}(R^4)$ term. With n_V gauge multiplets and n_H hypermultiplets, this is assured by the condition $n_H = n_V + 244$, which determines the number of hyper-multiplets in terms of the dimension of the $6D$ gauge group, $n_V = \mathrm{dim}G$ [14]. For simple gauge groups whose quartic casimir invariant is linearly independent of those at lower orders, there is another condition which amounts to requiring the vanishing of the $\mathrm{tr}(F^4)$ term.

If the only gauge group is the U(1) considered above, we have $n_V = 1$ and so $n_H = 245$ hyper-multiplets are required to cancel anomalies. (For our purposes more general gauge groups may also be possible, provided that the compactification solution we introduce in the next section remains a solution to the equations of motion.) In this case anomaly cancellation through a shift in the field B_{MN} requires the anomaly 8-form to be

$$I_8 = k\Big(\mathrm{tr}\, R^2 - vF^2\Big)\,\Big(\mathrm{tr}\, R^2 - \tilde{v}F^2\Big)\,, \tag{2.10}$$

where the trace is over the fundamental representation of $\mathrm{SO}(5,1)$ and multiplication represents the wedge product. Here the numbers k, v and $\tilde{v}$ are calculable given the precise fermion content of the $6D$ theory.

An anomaly of this form may be cancelled by adding the Green-Schwarz term

$$\mathcal{L}_{\mathrm{anom}} = -kvB\Big(\mathrm{tr}\, R^2 - \tilde{v}F^2\Big), \tag{2.11}$$

provided the transformation rule, eq. (2.3), for B_{MN} is modified to $\delta B = -\omega\, F + \alpha_L/v$ where $d\alpha_L = \delta\omega_L$ gives the transformation property of the gravitational Chern-Simons form, ω_L, which is in turn defined by the condition $d\omega_L = \mathrm{tr}\, R^2$. Invariance of the field strength, G_{MNP}, then requires its definition be modified to $G = dB + AF - \omega_L/v$.

The anomaly-cancelling term and the modifications to G are linked by supersymmetry to one another, and to other higher-derivative terms in the $6D$ action beyond those described above. For instance in the Einstein frame the U(1) gauge kinetic functions get modified to [18, 19]

$$-\,\frac{1}{4}\,\left(e^{-\phi} + \frac{\tilde{v}}{v}\,e^{\phi}\right)\,F_{MN}F^{MN}\,. \tag{2.12}$$

[2]If several bosonic fields are involved then the anomaly 8-form can be the sum of such products, one for each of the bosonic fields.

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As we see in more detail once the compactification to four dimensions is described below, all of these new terms are suppressed relative to the ones discussed in the previous sections in the sense that they involve higher powers of either $1/(e^\phi r^2)$ or e^ϕ/r^2 (or both).

2.3 The compactification

The equations of motion for the bosonic fields which follow from the action, eq. (2.4), are:

$$\begin{aligned} \Box\,\phi + \frac{1}{6}\,e^{-2\phi}\,G_{MNP}\,G^{MNP} + \frac{1}{4}\,e^{-\phi}\,F_{MN}F^{MN} - 2g^2 e^{\phi} &= 0 \\ D_M\Big(e^{-2\phi}\,G^{MNP}\Big) &= 0 \\ D_M\Big(e^{-\phi}\,F^{MN}\Big) + e^{-2\phi}\,G^{MNP}\,F_{MP} &= 0 \\ R_{MN} + \partial_M\phi\,\partial_N\phi + \frac{1}{2}\,e^{-2\phi}\,G_{MPQ}\,G_N{}^{PQ} + e^{-\phi}\,F_{MP}F_N{}^P + \frac{1}{2}\,(\Box\phi)\,g_{MN} &= 0\,. \end{aligned} \tag{2.13}$$

The compactification is found by searching for a solution to these equations which distinguishes four of the dimensions — $x^\mu, \mu = 0,1,2,3$ — from the other two — $y^m, m = 4,5$. The Salam-Sezgin solution is obtained by constructing this solution subject to the symmetry ansatz that the spacetime be separately maximally symmetric in the first four and last two dimensions. This leads to the following Freund-Rubin-type ansatz [20] for the solution: $\phi =$ constant and

$$g_{MN} = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & g_{mn}(y) \end{pmatrix} \qquad \text{and} \qquad F_{MN} = \begin{pmatrix} 0 & 0 \\ 0 & F_{mn}(y) \end{pmatrix}, \tag{2.14}$$

where $g_{\mu\nu}$ is a maximally-symmetric lorentzian metric (*i.e.* de Sitter, anti-de Sitter or flat space), and g_{mn} is the metric on the two-sphere, S_2. Maximal symmetry implies the gauge field strength is proportional to the sphere's volume form, ϵ_{mn}, and so

$$F_{mn} = f\;\epsilon_{mn}\,, \tag{2.15}$$

where f is a constant. All other fields vanish.

The gauge potential, A_m, which gives rise to this field strength is the potential of a magnetic monopole. As such, it is subject to the condition that the total magnetic flux through the sphere is quantized: $g\int_{S_2} B\,d^2y = 2\pi n$, with $n = 0, \pm1, \dots$. This requires the normalization constant, f, to be:

$$f = \frac{n}{2\,g\,r^2}\,, \tag{2.16}$$

where r is the radius of the sphere.

As is easily verified, the above ansatz solves the field equations provided that the following three conditions are satisfied: $R_{\mu\nu} = 0$, $F_{mn}F^{mn} = 8\,g^2 e^{2\phi}$ and $R_{mn} = -\,e^{-\phi}\,F_{mp}\,F_n{}^p = -f^2 e^{-\phi}\,g_{mn}$.[3] These imply the four dimensional spacetime is flat, the monopole number is $n = \pm 1$ and the sphere's radius is related to ϕ by

$$e^{\phi}\,r^2 = \frac{1}{4g^2}\,. \tag{2.17}$$

[3]In reference [14] the authors construct a similar solution by embedding the monopole in an E_6, also achieving flat four-dimensional space. The solution in [14] breaks supersymmetry.

Useful intuition about this result can be obtained by constructing the scalar potential for r and ϕ which is obtained by substituting our assumed background solution into the classical action. One finds in this way three contributions, coming from the Einstein-Hilbert term, the Maxwell kinetic term for A_m and the explicit dilaton potential. In order to eliminate mixing between these scalars and the fluctuations of the $4D$ metric, it is necessary to perform a Weyl rescaling to ensure the $4D$ Einstein-Hilbert action remains r-independent. We take, then:[4]

$$g_{MN} = \begin{pmatrix} r^{-2}\, g_{\mu\nu} & 0 \\ 0 & r^2 g_{mn} \end{pmatrix} \tag{2.18}$$

and find the following potential:

$$V = - \left. \frac{\mathcal{L}_B}{e_4} \right|_{\text{no derivatives}} = \frac{2\, g^2\, e^\phi}{r^2} \left(1 - \frac{1}{4g^2\, e^\phi r^2}\right)^2 , \tag{2.19}$$

where $e_4 = \sqrt{-\det g_{\mu\nu}}$. From this we see how eq. (2.17) emerges as the minimum of the scalar potential for r and ϕ. Because this potential is minimized at $V = 0$ we also see why the $4D$ metric must be flat. Finally, we see that the combination e^ϕ/r^2 parameterizes a flat direction, since its potential vanishes identically once $e^\phi r^2 = 1/4g^2$ has been chosen.

The existence of the flat direction parameterized by r^2/e^ϕ may be also inferred from the scaling symmetry of the supergravity equations of motion, eq. (2.8). Since $s := r^2/e^\phi$ transforms under this transformation while $t := e^\phi\, r^2$ does not, s plays the role of the dilaton for this symmetry. Since this scaling transformation is only a symmetry of the classical equations, and not of the action, the potential for s need not be exactly flat if we go beyond the classical approximation when computing the low-energy theory.

In the present case it happens that the flat direction with vanishing $4D$ cosmological constant is not lifted order-by-order in perturbation theory, as may be seen because the solution leaves one $4D$ supersymmetry unbroken. This may be seen by substituting the solution into the right-hand-side of eqs. (2.7) and checking that the result vanishes for a supersymmetry parameter which is independent of the 2D coordinates, y^m. Equivalently, spinors on S_2 which are constants are Killing spinors for this solution. Their existence is a consequence of the choice $n = \pm 1$ for the monopole number, since this ensures the cancellation of the gauge and spin connections in the covariant derivative, $D_\mu \varepsilon$ [10].

Some consistency conditions need be borne in mind if we regard this field configuration as a low-energy solution in string theory. In this case the approximation of weak string coupling requires we take $e^\phi \ll 1$ and the approximation of using a low-energy field theory similarly requires $r \gg 1$. Both of these requirements imply small values for the combination e^ϕ/r^2.

2.4 Low-energy fluctuations

Fluctuations about this background may be organized into four-dimensional fields according to the usual Kaluza-Klein procedure, with the generic mode having a mass which is at

[4] We implicitly change units when performing this rescaling, switching to the choice $\kappa_4^2 = 8\pi G_4 = 1$, rather than the same condition for the $6D$ quantity, κ_6^2.

least of order $1/r$. We wish to identify the effective four-dimensional theory which governs the physics below this scale.

2.4.1 Symmetries

As a preliminary to the identification of the light particle content of the $4D$ theory, we first identify how the background fields transform under the model's symmetries. Given the background fields of present interest — $g_{\mu\nu}, g_{mn}$ and F_{mn} — these are:

- Unbroken $4D$ Poincaré invariance, as given by the isometries of $4D$ Minkowski space:

$$\langle \delta g_{\mu\nu} \rangle = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0\,. \tag{2.20}$$

  These symmetries ensure the masslessness of the $4D$ graviton.

- Unbroken SO(3) invariance from the isometries of the internal metric,

$$\langle \delta g_{mn} \rangle = \nabla_m Z_n + \nabla_n Z_m = 0 \tag{2.21}$$

  which ensures the masslessness of three $4D$ spin-one particles.

- Broken local U(1) invariance, broken because of the transformation

$$\langle \delta B \rangle = \omega \langle F \rangle\,. \tag{2.22}$$

  From this we draw two conclusions. First, the $4D$ gauge field A_μ is not exactly massless, although it is systematically light compared to $1/r$ provided that the gauge coupling is small. Second, we see that in the Kaluza-Klein expansion $B_{mn}(x,y) = b\,\epsilon_{mn} + \dots$ (where ϵ_{mn} is the 2D volume form) the field b mixes with the Goldstone Boson for the U(1) gauge symmetry breaking.

- Unbroken $4D$ Kalb-Ramond symmetry

$$\langle \delta B_{\mu\nu} \rangle = \partial_\mu v_\nu - \partial_\nu v_\mu = 0 \tag{2.23}$$

  for constant v_μ.

- Because the 2-sphere's volume form, ϵ_{mn}, is harmonic, the action has a global symmetry, $\delta B_{mn} = c\,\epsilon_{mn}$, and this is superficially broken by the background, since $\langle \delta B_{mn} \rangle \neq 0$. However because $\langle F_{mn} \rangle = f\,\epsilon_{mn}$, there is a linear combination of this global symmetry and the U(1) gauge symmetry which is unbroken by the background fields:

$$\langle B_{mn} \rangle = c\,\epsilon_{mn} + \omega\,\langle F_{mn} \rangle = 0\,, \tag{2.24}$$

  provided $c = -f\,\omega$. This shows that in the Kaluza-Klein expansion $B_{mn} = b\,\epsilon_{mn}$, the field b becomes massless in the limit when either f or the U(1) gauge coupling vanish.

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2.4.2 Particle content

On symmetry grounds we expect the following bosonic particle content of the effective theory well below the scale $1/r$. (The corresponding fields are also given up to mixing due to the nonzero background flux $\langle F_{mn} \rangle$).

No.	Spin	Field
1	2	$g_{\mu\nu}(x)$
4	1	$A^a_\mu(x)$, 3 combinations of $g_{m\mu}(x,y)$
4	0	$B_{\mu\nu}(x)$, $\phi(x)$, $r(x)$, $B_{mn} = b(x)\,\epsilon_{mn}/e_2$

where we count here real scalar fields.

This counting arises as follows:

- The massless spin-2 particle follows as the gauge particle for the unbroken $4D$ Lorentz invariance of the background metric.
- The three massless spin-1 particles which arise as combinations of $g_{\mu n}$ are the gauge bosons for the SO(3) group of isometries of the 2-sphere.
- The field $B_{\mu\nu}$ dualizes to a massless scalar, a, according to the definition $\partial_\mu a = e^{-2\phi}\epsilon_{\mu\nu\lambda\rho}G^{\nu\lambda\rho}/e_4$. The symmetry $a \to a + \text{constant}$ can be broken by anomalies, which can arise after dualization due to the appearance of the Chern Simons terms in the field strength $G_{\mu\nu\lambda}$.
- The fields b and A_μ are *not* massless, but are systematically light compared to the scale $1/r$. The gauge field is not massless because the background field $F_{mn} \neq 0$ breaks the U(1) gauge symmetry, with an expectation value which is of order $f = \pm 1/(2gr^2)$ in size. The covariant derivative for b is $\partial_\mu b + fA_\mu$, indicating that b is the Goldstone boson which is eaten by the gauge boson. We shall see that the mass which is acquired in this way is suppressed by e^ϕ compared with $1/r$.[5]
- The combination $t := e^\phi r^2$ is also not massless since it is not a modulus of the background configuration, being fixed by the condition (2.17). We shall see that this scalar's mass is also suppressed by powers of e^ϕ and so can appear in the low-energy theory below $1/r$.
- The orthogonal combination $s := r^2 e^{-\phi}$ is massless in the classical approximation, as we saw from the scalar potential, eq. (2.19).

2.4.3 Light boson masses

To see why the masses of the fields A_μ and $t = r^2 e^\phi$ are suppressed by powers of e^ϕ, we must compute their kinetic terms in addition to their mass terms. For instance, for the gauge field, A_μ, the mass term arises from the square of the term $F_{mn}A_\mu$ which appears in

[5] The possibility that these fields get a mass appears to have been missed in ref. [10], but was recognised in ref. [14] in a similar context.

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the Kalb-Ramond kinetic term. Keeping in mind the Weyl rescaling of the $4D$ metric this gives a mass term of order

$$\frac{\mathcal{L}_{\rm mass}}{e_4\, e_2} = -\,\frac{1}{4} e^{-2\phi}\, G_{mn\mu}\, G^{mn\mu} \sim e^{-2\phi} F_{mn} F^{mn} A_\mu A^\mu \sim \frac{e^{-2\phi}}{g^2 r^2}\, A_\mu A^\mu\,, \tag{2.25}$$

where $e_2 = \sqrt{\det g_{mn}}$. By contrast, the kinetic term is

$$\frac{\mathcal{L}_{\rm kin}}{e_4\, e_2} = -\,\frac{1}{4}\, e^{-\phi}\, F_{\mu\nu} F^{\mu\nu} \sim r^4 e^{-\phi}\, F_{\mu\nu} F^{\mu\nu}\,. \tag{2.26}$$

Comparing these gives a gauge boson mass of order:

$$m_A^2 \sim \frac{e^{-\phi}}{g^2 r^6} \sim \frac{1}{g^2}\, \frac{1}{s\, t^2} \sim \frac{g^2}{s}\,, \tag{2.27}$$

where we have used the condition $g^2\, r^2\, e^\phi = O(1)$, $t = r^2\, e^\phi$ and $s = r^2/e^\phi$.

The mass for t is found in an identical way. The kinetic term for r arises from substituting the ansatz, eq. (2.18), into the $6D$ Einstein-Hilbert term. Together with the explicit ϕ kinetic term this leads to the following kinetic terms for t and s:

$$\begin{aligned} \mathcal{L}_{\rm kin} &= -g^{\mu\nu} \left[2\, \frac{\partial_\mu r\, \partial_\nu r}{r^2} + \frac{1}{2}\, \partial_\mu \phi\, \partial_\nu \phi \right] \\ &= -\frac{1}{4}\, g^{\mu\nu} \left[\frac{\partial_\mu s\, \partial_\mu s}{s^2} + \frac{\partial_\mu t\, \partial_\nu t}{t^2} \right]. \end{aligned} \tag{2.28}$$

In terms of s and t the potential, eq. (2.19), becomes

$$V = \frac{2\, g^2}{s} \left(1 - \frac{1}{4 g^2\, t} \right)^2\,, \tag{2.29}$$

and so $d^2V/dt^2\Big|_{\rm min} = 4\, g^2/(st^2)$. Comparing with the kinetic term gives a mass which is of the same order as was found above for m_A^2:

$$m_t^2 \sim \frac{g^2}{s} \sim \frac{g^2 e^\phi}{r^2}\,. \tag{2.30}$$

We see we are justified in keeping the fields t and A_μ in the effective theory so long as $g^2/s \ll 1$.

2.4.4 Light fermions

A similar calculation can be made for the spectrum of light fermions, and leads to the following light fermion spectrum:

No.	Spin	Field
1	3/2	$\psi_\mu(x)$
6	1/2	$\chi(x)$, $\lambda(x)$, 4 combinations of $\psi_m(x, y)$

For our purposes it is fruitful to determine how these fields assemble into multiplets of the unbroken $4D$ supersymmetry. The identification of these multiplets may be explicitly obtained by using the supersymmetry transformations of eq. (2.7) — such as by following the arguments of ref. [23] — and leads to the following:

- The massless gravitino required by the unbroken supersymmetry is the partner of the graviton.
- Three massless gauginos arise as partners of the SO(3) gauge bosons. These fermions come from the higher-dimensional gravitino due to the simultaneous existence of a Killing spinor and three Killing vectors.
- A massless fermion combines with s and a into a massless chiral multiplet, whose complex scalar part may be written $S = \frac{1}{2}(s + ia)$.
- Two fermions, with masses $m^2 \sim g^2/s$, join t and $A_\mu + \partial_\mu b/f$ to fill out a massive spin-1 multiplet. This massive multiplet can be regarded as the result of a massless spin-1 multiplet 'eating' the chiral multiplet whose complex scalar part is $T = \frac{1}{2}(t+ib)$ *via* the Higgs mechanism.

Once the fermions are chosen to transform in the standard way under $N = 1$ $4D$ supersymmetry, they do not carry the U(1) gauge charge, even though the $6D$ fermions did — c.f. eq. (2.1). In detail this happens because the $4D$ supersymmetry eigenstates are related to the $6D$ fermions by powers of the scalar e^{ib}, which cancel the $6D$ fermions' transformation properties. Only the gauginos of the low-energy theory transform nontrivially under the SO(3) gauge symmetry.

3. The $4D$ effective theory

Since the low-energy theory has an unbroken $N = 1$ supersymmetry, it must be possible to write it in the standard $N = 1$ supergravity form. From the previous section we see that the matter superfields in terms of which this action is expressed are the two chiral multiplets, S and T, the massive U(1) gauge multiplet, A, the three massless gauge multiplets, $A_a, a = 1, 2, 3$.

In order to completely specify all of the terms of the $4D$ supergravity action, it suffices to identify the Kähler function, $K(S, S^*, T, T^*, A, A_a)$, the gauge kinetic functions, $H_3(S, T)$ and $H_1(S, T)$ for the SO(3) and U(1) gauge groups, the superpotential, $W(S, T)$, and the Fayet-Iliopoulos term, ξz [24, 25].

3.1 The lowest-order action

In this section we determine these functions classically, by comparison with the direct truncation of the $6D$ action [22, 23], followed by a discussion of the kinds of corrections which may be expected for the result [23, 27].

3.1.1 The Kähler function

An important constraint on K arises because b is eaten by the U(1) gauge field, A_μ, since this implies its derivatives can only enter $\mathcal{L}$ through the gauge-invariant combination $\partial_\mu b + f A_\mu$. One infers from this that the superfields T and A must enter the Kähler function only through the combination $T + T^* + cA$, for a real constant c to be determined below. Similarly, the shift symmetry $a \to a+$ (constant) implies K can depend on S only through the combination $S + S^*$.

The form for K is most easily read off from the scalar kinetic terms, which in the Einstein frame must take the form $\mathcal{L}_{\rm kin} = -K_{ij^*}\partial_\mu z^i \partial^\mu z^{j*}$ for generic complex scalar fields, z^i. (As usual subscripts here denote derivatives of K with respect to the relevant scalar field, evaluated with all fields except the scalars vanishing.) Comparing this with the direct truncation calculation of the kinetic terms for r and ϕ, eq. (2.28), gives the result

$$K_{tr} = -\log\Big(S + S^*\Big) - \log\Big(T + T^* + cA\Big)\,. \tag{3.1}$$

3.1.2 The gauge kinetic functions

The gauge kinetic functions are more constrained than is the Kähler function since they must depend holomorphically on their arguments. They may be read off from the gauge boson kinetic terms, which must have the general form $\mathcal{L}_{\rm kin} = -\,\frac{1}{4}\,[({\rm Re}\;\, H_1)\, F_{\mu\nu}F^{\mu\nu} + ({\rm Re}\;\, H_3)\, F^a_{\mu\nu}F_a^{\mu\nu}]$. Alternatively, for some purposes they may be more simply obtained from the related terms $\mathcal{L}_\theta = -\,\frac{1}{4}\,[({\rm Im}\; H_1)\, F_{\mu\nu}\tilde{F}^{\mu\nu} + ({\rm Im}\; H_3)\, F^a_{\mu\nu}\tilde{F}_a^{\mu\nu}]$, since the imaginary parts of S and T appear in more restricted ways in the reduction of the $6D$ action.

Comparing these with the direct truncation of the $6D$ action gives the result: ${\rm Re}\, H_1 = e^{-\phi}\, r^2 = s = 2\;{\rm Re}\; S$, from which we find $H_1 = 2S$. (That the leading contribution to H_1 must be proportional to S follows from the recognition that H_1 scales like $H_1 \to \sigma^2\, H_1$ under the classical transformation, eq. (2.8), together with the transformations $S \to \sigma^2\, S$ and $T \to T$.) The higher-derivative corrections of eq. (2.12) which follow from anomaly cancellation correct this result to give

$$H_1 = 2\left(S + \frac{\tilde{v}}{v}\,T\right)\,. \tag{3.2}$$

A similar direct dimensional reduction for the SO(3) gauge fields is more involved, since the massless mode is a linear combination of the fields $A_\mu(x,y)$ and $g_{\mu n}(x,y)$ [26]. (The necessity for mixing between A_μ and $g_{\mu n}$ may be seen by performing a local SO(3) transformation corresponding to the general coordinate transformation $y^m \to \xi^m(x,y) = \omega^a(x)\, K^m_a(y)$, where $K^m_a(y), a = 1,2,3$ are the three Killing vectors which generate the SO(3) isometries of the sphere. Under this transformation the massless $4D$ gauge potential must transform as $\delta A^a_\mu = \partial_\mu\, \omega^a + \cdots$.) Consequently, the SO(3) gauge kinetic function acquires contributions from both the $6D$ Einstein-Hilbert and Maxwell terms of the action.

For our purposes the details of this reduction are not necessary in order to conclude that H_3 is given by an expression very much like eq. (3.2):

$$H_3 = 2(\alpha S + \beta T)\,, \tag{3.3}$$

for constants α and β which are given in terms of the anomaly coefficients k, v and $\tilde{v}$. This conclusion is most easily established by considering $\mathcal{L}_\theta$ and recognizing that for the SO(3) fields these terms are linear in $a = 2\,\mathrm{Im}\, S$ and $b = 2\,\mathrm{Im}\, T$. This is most easily seen from the contribution of the Lorentz Chern-Simons term in $G_{MNP}\, G^{MNP}$ and from the Green-Schwarz anomaly cancelling term, eq. (2.11). Linearity in a is as expected from the scaling property, eq. (2.8), together with the transformation properties of S and T.

3.1.3 The scalar potential

The constant c in the Kähler potential, the superpotential, W, and the Fayet Iliopoulos term, ξ, are fixed by considering the scalar potential, eq. (2.29). This must agree with the general supergravity form $V = V_D + V_F$, where

$$V_F = e^K \left[(K^{-1})^{ij^*} \, (W_i + K_i W)\, (W_j + K_j W)^* - 3|W|^2 \right],$$
$$V_D = -\frac{1}{2} \; (\mathrm{Re} \;\; H_1)\, D^2 - \frac{1}{2} \; (\mathrm{Re} \;\; H_3)\, D_a D_a \,, \tag{3.4}$$

where D_a and D are the auxiliary fields for the two factors of the gauge group, which we have not yet integrated out (hence the potential's unusual sign). Here, as usual, $(K^{-1})^{ij*}$ denotes the inverse of the matrix of second derivatives, K_{ij*}.

Given that neither S nor T carry SO(3) gauge quantum numbers, we see that $D_a = 0$ must be used when comparing with the truncated $6D$ action. Since T does transform under U(1), D can be nonzero and, from the Kähler and gauge kinetic functions found above, the U(1) D terms of the low-energy action arise from the following terms:

$$\begin{aligned} \mathcal{L}_D &= s \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D^2 \right) + D \left(\xi + \left. \frac{\partial K}{\partial A} \right|_{A=0} \right) \\ &= s \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D^2 \right) + D \left(\xi - \frac{c}{T + T^*} \right). \end{aligned} \tag{3.5}$$

Here ξ is the Fayet-Iliopoulos term, which is permitted only for U(1) gauge fields. Notice that consistency requires we use only the lowest-order expression $H_1 = 2S$ when comparing with the action given above.

Integrating out D implies the saddle-point condition

$$D = -\frac{1}{s} \, \left(\xi - \frac{c}{T + T^*} \right), \tag{3.6}$$

and so leads to the potential

$$V_D = +\frac{1}{2\, s} \, \left(\xi - \frac{c}{t} \right)^2 . \tag{3.7}$$

Comparing this with eq. (2.29) we read off:

$$\xi = \pm 2\, g \qquad \text{and} \qquad c = \pm \frac{1}{2\, g} \,. \tag{3.8}$$

This appearance of Fayet-Iliopoulos terms when the fermion content has a gauge anomaly is discussed in more general terms in ref. [28].

Since this completely accounts for the scalar potential and supersymmetry is unbroken, we conclude that the superpotential vanishes:

$$W = 0\,. \tag{3.9}$$

3.2 Perturbative corrections

The above expressions for K, W, H_1, H_3 and ξ are derived by classically truncating $6D$ supergravity, and so in principle they only apply strictly in the limit that $r \to \infty$ and $e^\phi \to 0$, since it is only in this limit that the corrections to truncation vanish. In this way we see that the truncation results are approximations to the full expressions which work for the region $s, t \to \infty$ of the space of moduli.

For sufficiently large s and t — both of which are large if $1/r^2 \ll e^\phi \ll 1$ — the corrections to the truncation may be computed order-by-order in a low-energy, weak-coupling expansion in powers of $1/s$ and $1/t$. (Some of these corrections have already been computed for the gauge kinetic functions above.) Fortunately, the interplay of $6D$ and $4D$ supersymmetry strongly restricts the form which such corrections may take [23, 27]. As usual, these implications are stronger for the holomorphic functions H_1, H_3 and W than they are for K and so we discuss these two cases separately.

3.2.1 Holomorphic functions

We first discuss the form which perturbative corrections may take for the holomorphic functions of the supergravity action. For the superpotential, W, as has been known for a long time [29], holomorphy completely forbids perturbative corrections from arising within perturbation theory [29], leading to the complete absence of correction to W to all orders in $1/T$ and $1/S$ [23, 27]. This leaves eq. (3.9) as the complete prediction to all orders.

Perturbative corrections to H_1 and H_3 do arise at one loop, and are given by the S-independent terms in eqs. (3.2) and (3.3). No further corrections beyond these are allowed to all orders in perturbation theory, however. This may be seen from the symmetry under shifts in $\mathrm{Im}\, S$, which is broken only by the Chern-Simons terms in the field strength for B_{MN}, since these determine the gauge transformation properties of B_{MN}, eq. (2.3). As we have seen, these Chern-Simons terms are themselves related to the Green-Schwarz action which cancels the gauge anomaly of the $6D$ fermions, and this connection with the anomaly precludes there being additional terms of this form which are generated beyond one loop. We see that expressions (3.2) and (3.3) are therefore the complete predictions — up to the additions of S- and T-independent constants — for H_1 and H_3 to all orders in $1/S$ and $1/T$.[6]

Perturbative corrections are necessarily concentrated into the Kähler function, and it is to a discussion of these that we now turn. These come in two forms.

3.2.2 Käher function: powers of $1/t$

Corrections to the Kähler function can arise, and do so independently as powers of $1/t$ and $1/s$, since these have roots in the more microscopic theory as independent expansions in powers of e^ϕ and $1/r$. This may be seen explicitly by considering two types of corrections to the lowest-order action in the $6D$ theory, as we now do. We start with powers of $1/t$,

[6] As has been noted elsewhere, since this argument relies on holomorphy it strictly applies only to the Wilson action, and not necessarily to the generator of 1PI vertices [30, 31].

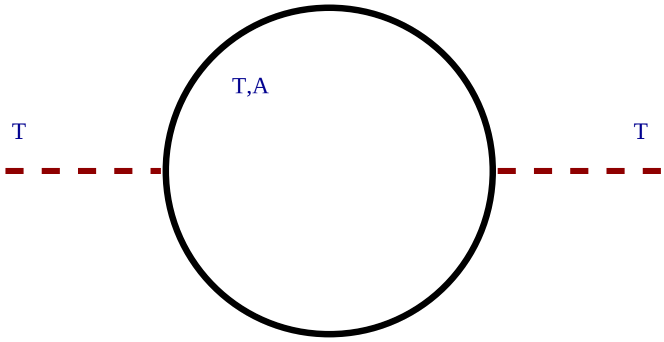

Figure 1: One-loop diagram contributing to the kinetic term of the field T. The internal lines are also T and/or the U(1) gauge field.

which play an important role in what follows, and which we now argue correspond to the contributions under dimensional reduction of higher-derivative corrections to the $6D$ effective theory.

The simplest way to identify corrections to K is to compute the corrections to the Kähler metric by examining the kinetic terms of the scalars r and ϕ in the $4D$ effective action. Examples of higher-derivative corrections to these kinetic terms are the contributions of higher-curvature terms to the radion kinetic energy. For instance a higher-curvature correction to the Einstein-Hilbert action (in the string frame) in six dimensions

$$\frac{\mathcal{L}_{SF}}{e_6} \sim e^{-2\phi}\Big[R_s + k_n R_s^n\Big] \tag{3.10}$$

becomes, in the Einstein frame

$$\frac{\mathcal{L}_{EF}}{e_6} \sim R + k_n e^{-(n-1)\phi} R^n \,, \tag{3.11}$$

due to the rescaling $g_{MN} \to e^{\phi} g_{MN}$ which is required to remove ϕ from in front of the Einstein-Hilbert part of the action. On dimensionally reducing we extract one factor of the $4D$ Ricci tensor, $R_{\mu\nu} \propto \partial_\mu r \, \partial_\nu r / r^2$, from R^n, with the remaining factors being proportional to the two-dimensional curvature: $R_{(2)}^{n-1} \propto (1/r^2)^{n-1}$, leading to the $4D$ kinetic term

$$\frac{\mathcal{L}_{\rm kin}}{e_4} \sim \frac{\partial_\mu r \, \partial^\mu r}{r^2}\left[1 + \frac{k_n}{(e^{\phi} r^2)^{n-1}}\right] \sim \frac{\partial_\mu r \, \partial^\mu r}{r^2}\left[1 + \frac{k_n}{t^{n-1}}\right]. \tag{3.12}$$

For instance, in string theory such corrections could arise from sigma-model corrections at string tree level.

3.2.3 Käher function: powers of $1/s$

Powers of $1/s$ can arise due to loops within the $4D$ theory itself. For instance, consider the one-loop correction to the kinetic term for T which is induced by the graph of figure 1. Since the $4D$ loop integrals diverge (quadratically) in the ultraviolet, they are insensitive to the masses of the multiplets in the loop and we can ignore the mixing between the T multiplet and the U(1) gauge multiplet. If we take all internal lines in figure 1 to be T fields, then the vertices are of order $\partial^3 K/\partial t^3 \sim 1/t^3$, while each propagator contributes $[K_{TT^*}(p^2+m_t^2)]^{-1}$ with $K_{TT^*} \sim 1/t^2$. Taking the $4D$ ultra-violet cutoff to be $M_c^2 \sim 1/r^2 \sim M_p^2/\sqrt{st}$, we estimate:

$$\delta K_{TT^*}(T \text{ loop}) \sim \frac{M_c^2}{(4\pi)^2 M_p^2} \left(\frac{1}{t^3}\right)^2 \left(t^2\right)^2 \sim \frac{1}{(4\pi)^2\, t^2 \sqrt{st}}\,. \tag{3.13}$$

Alternatively, if one of the internal lines of figure 1 is a U(1) gauge multiplet, then the coupling between V and T, given by the lowest order Kähler function $K = -\log(T+T^*+cV)$, is of order $\partial^3 K/\partial V\,\partial T \partial T^* \sim c/t^3$, and we see that each vertex of figure 1 contributes a factor c/t^3. Taking $s \gg t$ the gauge propagator contributes a factor of $1/s$, so we estimate:

$$\delta K_{TT^*}(T - V \text{ loop}) \sim \frac{M_c^2}{(4\pi)^2 M_p^2} \left(\frac{c}{t^3}\right)^2 \left(\frac{t^2}{s}\right) \sim \frac{1}{(4\pi)^2\, t^3 s \sqrt{st}}\,, \tag{3.14}$$

where we use the lowest-order condition $t \sim 1/g^2 \sim c^2$.

3.2.4 Käher function: logarithms of $1/s$

Having seen how powers of $1/t$ and $1/s$ control the modifications to K, we next consider the possibility that more subtle types of corrections arise, which depend logarithmically on $1/s$. Indeed, logarithmic dependence on coupling constants is known to arise in $4D$ physics if the energies of some low-lying states, E_l, are suppressed by powers of coupling constants, g, relative to higher-energy states, E_h: $E_l \propto g^n E_h$. In this case logarithms of couplings can arise as logarithms of energy ratios: $\log(E_h/E_l) \sim n\,\log(1/g)$. The most well-known example of this type is perhaps the QED prediction for the Lamb shift, which involves a famous factor of $\log(1/\alpha)$ [32]. The potential for these kinds of logarithms exists in the low-energy $4D$ theory arising from the $6D$ supergravity compactification considered above because the KK states have mass of order the compactification scale, $m_{KK}^2 \sim 1/r^2 \sim M_p^2/(st)^{1/2}$, while the U(1) gauge multiplet has a mass which is further suppressed by powers of t/s and g: $m_t^2 \sim g^2 e^{\phi}/r^2 \sim g^2\, M_p^2/s = g^2 m_{KK}^2 (t/s)^{1/2}$.

Because these logarithms can be traced to the appearance of large logarithms in the running of the couplings in four dimensions, their appearance can be understood (and often re-summed) using standard renormalization-group arguments [33, 34]. To this end imagine running the $4D$ effective theory from its ultraviolet cutoff, M_c, down to $m_t \ll M_c$. The running of the inverse couplings, $H_1(S,T)$ and $H_3(S,T)$, in this interval is given by

$$\begin{aligned} H_1(S,T)|_\mu &= H_1(S,T)|_{\mu_0} + b_1\,\log\left(\frac{\mu^2}{{\mu_0}^2}\right) \\ H_3(S,T)|_\mu &= H_3(S,T)|_{\mu_0} + b_3\,\log\left(\frac{\mu^2}{{\mu_0}^2}\right), \end{aligned} \tag{3.15}$$

where b_1 and b_3 are the standard supersymmetric one-loop beta-function coefficients for the U(1) and SO(3) gauge groups, respectively. Using the expressions (3.2) and (3.3), we may solve for the running of s and t

$$s(\mu^2) = s_0 + b_s\,\log\left(\frac{\mu^2}{\mu_0^2}\right), \qquad \text{and} \qquad t(\mu^2) = t_0 + b_t\,\log\left(\frac{\mu^2}{\mu_0^2}\right), \tag{3.16}$$

where, as long as the equations are non-singular, b_s and b_t are linear combinations of b_1 and b_3 depending on the coefficients of S and T in H_1 and H_3.

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The dependence of large logarithms on m_t may then be traced by running these $4D$ couplings down to $\mu = m_t$ from some higher scale, like $\mu_0 = m_{KK}$. This gives large logarithms of the form

$$\log\left(\frac{m_{KK}^2}{m_t^2}\right) \sim \log\left(\frac{1}{g^2}\sqrt{\frac{s}{t}}\right) + \text{constant}\,. \tag{3.17}$$

We see from this that the existence of logarithms of s and t in the corrections to K are quite likely.

Without performing a more sophisticated calculation it is difficult to pin down the precise power of s and/or t which appears inside the logarithm. This is because low-energy logarithms like $\log(m_{KK}^2/m_t^2)$ can in principle combine with other large logarithms which arise purely from the high-energy theory, such as $\log(M_p^2/m_{KK}^2)$ to give logarithms which compare the small mass m_t with some other high-energy scale like M_p:

$$\log\left(\frac{M_p^2}{m_t^2}\right) \sim \log\left(\frac{s}{g^2}\right) + \text{constant}\,. \tag{3.18}$$

We therefore parameterize this possibility by writing the resulting full RG-improved Kähler function as

$$K = -\log\left[s - b_s \log\left(st^a\right) + k_s\right] - \log\left[t - b_t \log\left(st^a\right) + k_t + cA\right], \tag{3.19}$$

where a, k_s and k_t are order-unity constants.

Notice that expanding eq. (3.19) in powers of $1/s$ and $1/t$ gives the corrections to K to leading order in $1/t$ and $1/s$, but to all orders in $(1/s)\log(M_c^2/m_t{}^2)$ and $(1/t)\log(M_c^2/m_t^2)$, if $M_c \sim m_{KK}$ is the ultraviolet matching scale. This observation will become important later when we find minima for the scalar potential.

3.3 Nonperturbative effects in $4D$

Given the above semiclassical approximation to the functions K, H, H_{ab} and W, we may use general knowledge of $4D$ $N = 1$ supersymmetric theories to understand the physics at energies much below the compactification scale. In particular, our interest is in the existence of any other mass scales at very low energies which might lift the degeneracy of the flat direction described by S.

It is useful to re-instate the Planck mass and to identify the mass scales which arise in the low-energy $4D$ supergravity. These are:

- The $4D$ Planck mass: $M_p^2 = 1/\kappa_4^2 = 1/(8\pi G_4)$, as defined by the $4D$ graviton couplings.
- The $4D$ cutoff: $M_c \sim 1/r \sim M_p/(st)^{1/4}$, which defines the scale above which the theory is no longer efficiently described by a $4D$ lagrangian.
- $m_D \sim gM_p/\sqrt{s} \sim gM_c(t/s)^{1/4} \sim M_c^2/M_p$, the semiclassical mass for the U(1) gauge-boson supermultiplet. (The last approximate equality here uses the semiclassical condition $t \sim 1/g^2$ found earlier.)

To these semiclassical mass scales should be added a new, nonperturbative one: $\Lambda \sim \mu \exp[-(\nu s(\mu) + \lambda\, t(\mu))/3]$, where ν and λ are positive constants which are related to the renormalization-group coefficients for s and t by the condition that Λ be independent of renormalization point μ. Using eqs. (3.16) this implies

$$\nu\, b_s + \lambda\, b_t = \frac{3}{2}\,. \tag{3.20}$$

This new scale arises because the low-energy theory's SO(3) gauge theory is asymptotically free, with Λ defining the confinement scale where its effective coupling becomes strong. At this scale the gauginos of the SO(3) theory condense [31], and because of this condensation (together with the absence of matter fields carrying approximate global chiral symmetries) the SO(3) gauge sector acquires a gap in its spectrum which is of order Λ. The massive energy eigenstates which result are the SO(3)-singlet bound states of the gluons and gluinos.

As is well known, this condensation dynamically generates a superpotential in the low-energy theory [2, 31], which is of order Λ^3:

$$W = w_0 \; \exp[-\nu\, S - \lambda\, T]\,, \tag{3.21}$$

for some constant $w_0 \sim \mu^3$.

This superpotential contributes to the scalar potential for s and t by generating a nonzero V_F, which was absent semiclassically. It is this new term which is responsible for the qualitatively new features of the low-energy theory: the lifting of the flat direction for s.

4. Dynamics of the flat directions

We have seen that the strongly-coupled SO(3) gauge couplings dynamically generate a superpotential at low energies, and so the two terms, V_D and V_F conflict in what they would like the fields t and s to do. The semiclassical term, V_D, is minimized when $t \sim 1/g^2$, while the nonperturbative term, V_F, is minimized when $t \to \infty$. These cannot be simultaneously minimized and so a compromise must be struck for which at least one of V_F or V_D is nonzero. We find that the vacuum to which this competition between V_F and V_D leads depends in a crucial way on the form of the corrections to K discussed above.

4.1 Dilaton runaway

As a first approximation to the shape of this potential, we consider the superpotential, eq. (3.21), but ignore all corrections to the leading semiclassical Kähler function, eq. (3.1), and gauge kinetic functions. This leads to a scalar potential of the form $V = V_D + V_F$, where V_D may be read off from eq. (2.29), and V_F is given by:

$$V_F(s,t) = \frac{|w_0|^2}{st}\, e^{-\nu\, s - \lambda\, t} \; \Big[(1+\nu\, s)^2 + (1+\lambda t)^2 - 3 \Big]\,. \tag{4.1}$$

If $\Lambda \ll M_p$ we have $w_0 \ll 1$ and the minimum for t is close to the zero of V_D: $1/t = 4g^2 + O(|w_0|^2)$. To linear order in $|w_0|^2$ the potential for s then becomes $V_{\rm eff}(s) \approx V_F(1/t = 4g^2)$, and so

$$V_{\rm eff}\,(s) = \frac{4g^2\,|w_0|^2}{s}\, e^{-\nu\, s - \lambda/(4g^2)} \, \left[(1+\nu\, s)^2 + \left(1 + \frac{\lambda}{4g^2}\right)^2 - 3\right]. \tag{4.2}$$

We find that the potential for s which is generated in this approximation does not have any minima for positive s besides the runaway solution for which $s \to \infty$. This is the familiar dilaton runaway, with the SO(3) gauge coupling generically driven to zero as s runs off to infinity.

4.2 Dilaton stabilization

The weak part of the previous analysis is the use of the lowest-order Kähler function, eq. (3.1), despite using a nonperturbative expression for the superpotential. We now show that using the renormalization-group-improved expression, eq. (3.19), can generate a potential for s which can have other minima besides the dilaton runaway.

We begin with the Kähler function,

$$\begin{aligned} K(s,t) &= -\log\left[s + \frac{b_s}{2}\log\left(\frac{st^a}{q}\right)\right] - \log\left[t + \frac{b_t}{2}\log\left(\frac{st^a}{q}\right)\right] + \\ &\quad + O\left(\frac{\log(st^a/q)}{s^2}, \frac{\log(st^a/q)}{t^2}\right), \\ &\equiv -\log s_0 - \log t_0 + O\left(\frac{\log}{s^2}, \frac{\log}{t^2}\right), \end{aligned} \tag{4.3}$$

where a and q are constants, and where s_0, t_0 are the fields evaluated at the high scale. Notice that changing from s_0, t_0 to s, t in K (and then constructing the scalar potential V) is *not* simply the same as performing a trivial change of variables on the potential V itself. It is not, because this change is not a holomorphic redefinition of S and T.

Under the assumption that $|w_0| \ll 1$ we may compute the effective potential as before, by first minimizing $V_D(s,t)$ to obtain $t = t(s)$ and then examining $V_{\rm eff}(s) \approx V_F[s, t(s)]$. The minimum of V_D occurs when

$$K_T + \epsilon = 0\,, \tag{4.4}$$

where ϵ is a constant which is of order g^2 and

$$K_T = \frac{\partial K}{\partial T} \approx -\,\frac{1}{t_0} - \frac{a}{2}\left(\frac{1}{t}\frac{b_t}{t_0} + \frac{1}{s}\frac{b_s}{s_0}\right). \tag{4.5}$$

In the case where $a = 0$ (4.4) can be solved analytically, so that V_D is minimized for $t(s)$ satisfying

$$t_0 \equiv t + \frac{1}{2}\, b_t \log\left(\frac{s}{q}\right) \approx \frac{1}{\epsilon}\,. \tag{4.6}$$

Solving this for $t(s)$ and using the result in $V_F[s,t(s)]$ gives $V_{\rm eff}(s)$. The computation of V_F requires the inverse matrix:

$$\begin{aligned}
(K^{-1})^{SS^*} &= \frac{K_{TT^*}}{||K||}\,, \\
(K^{-1})^{TT^*} &= \frac{K_{SS^*}}{||K||}, \\
(K^{-1})^{ST^*} &= (K^{-1})^{TS^*} = -\frac{K_{ST^*}}{||K||}\,,
\end{aligned} \tag{4.7}$$

where $||K||$ is the determinant of the matrix K_{ij^*}. To this order in the Kähler function we may ignore the difference between s and s_0 and t and t_0 after taking derivatives, so that

$$\begin{aligned}
K_{TT^*} &= \frac{1}{t_0^2} \\
K_{SS^*} &= \frac{1}{s_0^2}\left(1+\beta_t+{\beta_t}^2+3\beta_s+{\beta_s}^2\right) \\
K_{ST^*} &= \frac{\beta_t}{s_0 t_0} \\
||K|| &= \frac{1}{s_0^2\,t_0^2}\left(1+\beta_t+3\beta_s+{\beta_s}^2\right),
\end{aligned} \tag{4.8}$$

where $\beta_s = \frac{1}{2}b_s/s_0$, $\beta_t = \frac{1}{2}b_t/t_0 \equiv \frac{1}{2}\epsilon b_t$. We also have the Kähler derivatives

$$\begin{aligned}
D_S W &= W_S + K_S\, W \\
&= -\left[\nu + \frac{\sigma}{s_0}\right] W \\
D_T W &= W_T + K_T\, W \\
&= -\,(\lambda+\epsilon)\, W\,,
\end{aligned}$$

where both results are evaluated at $t=t(s)$ and we have used (3.20), and $\sigma = 2(7/8+\beta_t+\beta_s)$.

One finds in this way the expression for $V_{\rm eff}(s) = V_F[s,t(s)]$:

$$V_{\rm eff}(s) = \frac{|w_0|^2}{s_0 t_0}\, N(s_0,t_0)\, e^{-2\nu s_0 - 2\lambda t_0}\,, \tag{4.9}$$

where

$$N(s_0,t_0) = \frac{(\nu s_0+\sigma)^2 - 2B\,(\nu s_0+\sigma) + C\,t_0^2\,(\lambda+\epsilon)^2}{1+\beta_t+3\beta_s+\beta_s^2} - 3 \tag{4.10}$$

with

$$B = \beta_t t_0(\lambda+\epsilon) \qquad \text{and} \qquad C = 1+\beta_t+\beta_t^2+3\beta_s+\beta_s^2\,. \tag{4.11}$$

Notice that this reduces to the previous runaway potential in the limit $b_t \to 0, b_s \to 0$, as long as we also set $\sigma = 1$ (the conditions (3.20) do not apply in this limit). β_s and σ are both s_0-dependent quantities.

This potential is drawn in figure (2) for the choice, in (3.3), $\alpha = \beta = 1/800$, and with $\tilde{v}/v = -9/4$, $q = 100$, $b_1 = -1/10$, $b_3 = 6/(4\pi)^2$, $\nu = 0.005$ and $\epsilon = .28$. λ is determined

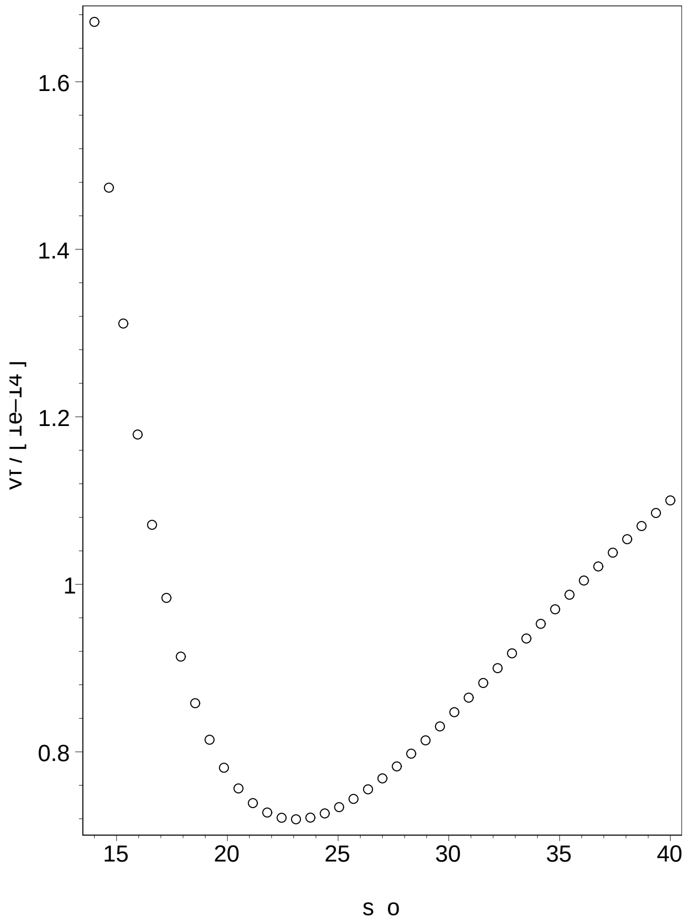

Figure 2: The effective potential for s computed using the renormalization-group improved potential. Parameters are chosen as described in the main text.

in terms of these by the condition $\nu\, b_s + \lambda\, b_t = 3/2$, while b_t and b_s are determined from b_1 and b_3. For these choices the potential is minimized by the field values $s = 34$ and $t = 8.6$, corresponding to $t_0 = 1/\epsilon \approx 3.6$ and $s_0 = 23$.

It remains to be shown that values this small for α and β can be obtained from realistic string models. The above discussion nonetheless suffices to make our main point that the renormalization-group-improved Kähler function can produce nontrivial minima for the modulus s. We have also found minima having larger values of α and β ($|\alpha|, |\beta| \sim O(0.1)$), albeit for small values of s and t which lie at the limit of what can be understood perturbatively in powers of $1/s$ and $1/t$. The generic existence of such minima can be seen by observing that as long as the potential has a maximum, and increases as one approaches the origin ($s = t = 0$) from the right, then a minimum must exist in between (barring the existence of new singularities in this regime). In the present case of the larger values of α and β, maxima exist for s and t well within the perturbative regime, allowing us to infer the existence of the minima at much smaller field values: $s, t \sim O(0.1)$.

The quantities parameterizing the strength of the supersymmetry breaking are given by the *vev* of the potential, $V_{\rm eff}(s)|_{\rm min}$, as well as the expectation values of the auxiliary

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fields:

$$\begin{aligned} F_i &= e^{K/2} D_i W \,, \\ M &= e^{K/2} \frac{W}{3} \,, \end{aligned} \tag{4.12}$$

where i runs over S and T. The mass of S is approximately given by

$$m_s^2 \approx e^{-K} \left. \frac{d^2 V_{\rm eff}(s)}{ds^2} \right|_{\rm min} \,. \tag{4.13}$$

For the minimum described numerically above, we find $\langle V \rangle \approx 7 \times 10^{-15}$, $V'' \approx 9 \times 10^{-17}$, $F_s \approx -9 \times 10^{-8}$, $F_t \approx -2 \times 10^{-6}$, $M \approx 2 \times 10^{-7}$ and $m_s^2 \approx 7 \times 10^{-15}$, all in Planck units. The supersymmetry-breaking scale is therefore seen to be quite low, compared to, say, the compactification scale $M_c \sim M_p/(st)^{1/4} \sim M_p/3$.

5. Discussion

We now summarize our results, and outline some of their potential applications.

5.1 Summary

We have revisited the Salam-Sezgin compactification of gauged $N = 1$ $6D$ supergravity, and have computed the $N = 1$ $4D$ supergravity to which it leads at low energies. The low-energy field content to which we are led is supergravity coupled to a supersymmetric $\mathrm{U}(1) \times \mathrm{SO}(3)$ gauge theory plus several chiral multiplets which describe the compactification's moduli. The low-energy theory has the following properties:

- The U(1) multiplet 'eats' one of the chiral multiplets *via* the Higgs mechanism at the classical level giving these fields masses which are nonzero but systematically light compared to the compactification scale. The corresponding scalar potential arises in $4D$ through a Fayet-Iliopoulos term for the U(1) gauge group.

- One combination of scalars parameterizes a flat direction which remains massless to all orders in a semiclassical expansion, and supersymmetry remains unbroken along this flat direction.

- The nonabelian SO(3) gauge multiplet is asymptotically free and once its coupling becomes large its gauginos condense and generate a nonzero superpotential. The scalar potential which results from this superpotential competes with the U(1) D-term potential and lifts the degeneracy of the flat direction.

- The vacuum to which the theory tends depends on the precise form of the perturbative corrections to the Kähler potential. Using the lowest-order result, $K = -\log s + \cdots$, leads to the standard dilaton runaway, in which the massless field parameterizing the flat direction runs off to infinity. In this limit the SO(3) gauge coupling vanishes and supersymmetry remains unbroken.

- If we instead use the renormalization-group-improved version of K, then the runaway can be stabilized for some choices of the parameters. In this context the significance of the present analysis is to identify a type of logarithmic dependence of K on s which would be sufficient to stabilize the runaway. Of course, we do not know yet whether the required parameters can actually arise for low-energy perturbations about a real string vacuum. However, we regard the potential rewards of their discovery to provide sufficient motivation for taking a proper look.

At first sight, this last item appears to run contrary to a standard argument by Dine and Seiberg against the possibility of fixing the dilaton at weak string coupling [36]. This argument essentially states that if the potential is a series in $1/s$, then any minimum — besides the runaway $s \to \infty$ — must balance different terms in this series against one another, and so be incalculable within the context of perturbation theory.

Despite this, we are able to find nontrivial minima in our analysis for two reasons. First, given that t and s are large at the minimum, but $(1/t)\log s$ is $O(1)$, we see that the Dine-Seiberg argument is correct inasmuch as it states that the potential is required to all orders in $(1/t)\log s$ in order to determine its minima. Fortunately, this form is known by virtue of the renormalization-group re-summation.

Second, one may ask how t and s could be large at the minimum in the first place if there are no large parameters in the potential. Although we have not exhaustively searched parameter space for other solutions, it appears that we only obtain large values for s and t when we choose small values for the parameters α and β, and so this may explain the origin of the nontrivial minima within perturbation theory. To the extent that the appearance of these extra parameters which can be tuned to get weak coupling are required, our analysis would be similar to the older racetrack scenarios.

In the end, it may be that realistic string models do not provide α and β of the required magnitude. We regard the artifice of exploring the consequences of their being small nonetheless to be of some value, because it allows us to infer some evidence for the existence of minima in the strong-coupled regime even in the cases where α and β are larger. In this case the minima we find would be pushed into the strong-coupling region for which our calculational methods do not directly apply. Nevertheless the existence of these minima still follows from the existence of a maximum for larger values of s and t, together with the general property that the potential is positive and diverging as $s, t \to 0$. Indeed, the existence of the maxima within the perturbative regime can be inferred for a wider range of values for α and β than can the existence of the minima, leaving only the existence of the singularity of the potential at small s, t to be established using more robust arguments.

5.2 Potential applications

Both the runaway and stabilized scenarios have several potentially interesting applications to low-energy phenomenology and to cosmology, which we now briefly outline.

5.2.1 Inflation

The most conservative application of the above dynamics only relies on the semiclassical stabilization of the T modulus through the U(1) D-terms, and does not use the dilaton stabilization mechanism. This application is to efforts to model inflation within the brane-world [5, 6], and relies on the recently-made observation [6] that in some circumstances supersymmetry can help make inflation easier to obtain if it is being driven by brane physics.

A generic problem with obtaining exponential inflation from brane physics arises because of the dynamics of the breathing mode of a compactification. That is, even if an approximately-constant potential energy can be found for a candidate inflaton which lives on a particular brane, this potential energy tends not to lead to de Sitter style inflation. It does not do so because a constant energy on a brane is typically really constant only in the Jordan frame tailored to the brane, and not in the Einstein frame for which the Planck mass is constant. Instead, in the Einstein frame it appears as a radius-dependent source of energy and so it causes the sizes of the extra dimensions to evolve rather than causing the ordinary 4 large dimensions to inflate.

Successful inflation along these lines therefore also requires the extra dimensions to be stabilized, and it turns out that inflation can be more easily obtained if it is a combination like $s = r^2/e^\phi$ or $t = r^2 e^\phi$ which is fixed by the stabilizing physics, rather than if r or ϕ is separately fixed [6]. The semiclassical $6D$ model presented here is precisely such a stabilization mechanism for t, and so suggests that a similar mechanism might be employed to generate an inflationary brane scenario.

Furthermore, our result of getting a positive cosmological constant after fixing the moduli can be directly used as the starting point of D-brane inflation along the lines of references [5], where one imagines also adding an attractive potential which causes the separation between D-branes to be the inflaton field after all closed string moduli have been fixed.

5.2.2 Supersymmetry breaking

If the runaway is stabilized the effective $4D$ model dynamically breaks supersymmetry at a scale which can be naturally very small compared with the compactification scale. If electroweak symmetry breaking occurs at the supersymmetry-breaking scale, and if fundamental scales like M_s and M_c are chosen near the Planck or GUT scales, then this would provide a Kaluza-Klein realization for using dynamical supersymmetry breaking to naturally generate the electroweak gauge hierarchy, along lines initially proposed some time ago [37].

The low-energy implications of such a model may be inferred by regarding the entire theory considered here to be the hidden supersymmetry-breaking sector to which standard-model particles are coupled [38]. As is easily verified, the large hierarchy $s \gg t$ which the stabilization mechanism predicts ensures that the auxiliary field for S is the largest supersymmetry-breaking *v.e.v.*. This makes the phenomenological implications of this kind of supersymmetry breaking the same as for a dilaton-dominated scenario. This has the virtue of being among the most predictive kinds of string-motivated supersymmetry-breaking scenarios, with definite relations predicted for the spectrum of superpartners [38].

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5.3 String theory derivation?

It is an interesting challenge to derive the $6D$ theory we started with from string theory. First we note that typically $6D$, $N = 2$ supergravity is obtained from K_3 compactifications of type I or heterotic strings. However the supergravity theory obtained in this way is ungauged, whereas ours is gauged supergravity that includes a nontrivial potential $\sim ge^{-\phi}$.

Recently it has been realized that massive 10D and gauged supergravities in lower dimensions can be obtained either from string compactifications on spheres [40][7] or toroidal and related compactifications, such as K_3, in the presence of RR or NS-NS fluxes. In the latter case, typically a p-form F_p integrated around non-trivial cycles of the compact space Γ_p can be different from zero, $\int_{\Gamma_p} F_p \neq 0$. The form F_p can be expanded in terms of harmonic forms ω_p^i:

$$F_p = \nu^i \omega_p^i \,, \tag{5.1}$$

where the coefficients ν^i will correspond to the fluxes. In particular these fluxes give rise to potentials precisely of the form we started with, the flux being identified with the gauge coupling constant of the effective gauged supergravity theory. In this way several maximal gauged supergravity theories have been obtained from toroidal compactifications with fluxes [41].

Also, compactifications on $K_3 \times T_2$ have been considered with fluxes in both K_3 and T_2. Furthermore, fluxes have also been shown in type IIB and M theory to freeze geometric moduli [42], just as the T field in the present model is frozen.

We have to recall that the general $N = 2$ $6D$ supergravity has a more complicated spectrum of scalar fields than the one we used, since the gauge group can be much larger than the simplest U(1) that we considered [43]. In that case the potential is quadratic in the hypermultiplet scalars, with overall factors of $e^{-\phi}$, a natural outcome of the fluxes in string theory. It is not completely clear to us how to derive precisely the Salam-Sezgin action from this class of backgrounds yet, although it looks very suggestive.

Furthermore, string compactifications on spheres and related geometries have also been successful in deriving gauged supergravities. In particular the maximal $6D$ supergravity was derived from an S^4 string compactification and a detailed comparison of the potential was achieved with the $N = 4$ gauged supergravity of Romans [44].[8] We leave as an open question the possible derivation of our $N = 2$ action from this construction as well as any possible relationship with the K_3 backgrounds with fluxes. If such a construction is found then it will be interesting since it will give rise to realistic string compactifications on manifolds which are not Ricci flat (since at least the 2-sphere is not), contrary to standard beliefs.[9]

[7] We thank C. Pope for many discussions on these points.

[8] After completing this work it was pointed out to us that fluxes over $\mathcal{P}^1(\mathcal{C}) = S^2$ were considered in ref. [45] in Heterotic and type-II compactifications.

[9] After finishing this article we became aware of earlier [42] and new [46] work presenting examples of this type. It may be intersting to unravel any connection between these constructions and our work.

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5.4 Open questions

There are many questions left open at the moment. Any application of the dilaton stabilization mechanism would be on a much better foundation if the corrections of the form required for K could be computed as the consequence of an actual microscopic underlying theory, such as from a viable string configuration. We do not know how to obtain such a vacuum, but regard the identification of the features required to enable stabilization as a worthwhile first step towards identifying what would be required of an underlying model in order to reproduce this stabilization.

In general terms we see this model as a toy laboratory for understanding more complicated string compactifications. It would be interesting to have an explicit construction that precisely generates this model, in which case it would be promoted to become a consistent string vacuum. Nevertheless we believe that many of the properties we discuss here apply more generally than just to this particular model.

Another open question concerns the introduction of branes and antibranes into the model, perhaps along the lines of [39], potentially leading to an implementation of a $6D$ brane-world scenario. The concreteness of the construction has the virtue of allowing the explicit study of their implications for supersymmetry breaking and cosmology.

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